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Chinese Journal of Aeronautics 21(2008) 472-480
Chinese Journal of Aeronautics
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A Back-stepping Based Trajectory Tracking Controller for a Non-chained Nonholonomic Spherical Robot

Zhan Qiang*, Liu Zengbo, Cai Yao

Robotics Institute, Beijing University of Aeronautics and Astronautics, Beijing 100191, China

Received 27 May 2008; accepted 17 July 2008

Abstract

Spherical robot has good static and dynamic stability, which provides it with strong viability in hostile environment, but the lack of effective control methods has hindered its application and development. This article deals with the dynamic trajectory tracking problem of the spherical robot BHQ-2 designed for unmanned environment exploration. The dynamic model of the spherical robot is established with a simplified Boltzmann-Hamel equation, based on which a trajectory tracking controller is designed by using the back-stepping method. The convergence of the controller is proved with the Lyapunov stability theory. Numerical simulations show that with the controller the robot can globally and asymptotically track desired trajectories, both linear and circular.

Keywords: spherical mobile robot; trajectory tracking control; back-stepping; Lyapunov function

1 Introduction

In recent years, the spherical robot as a member of the new type of mobile robots has made its debut[1-7]. It consists of a ball-shaped outer shell to accommodate the whole mechanism inclusive of control devices and energy sources. The robot can fulfill controllable movement on the basis of the principles of gravity center offset and angular momentum conservation. Characterized by good dynamic and static stability as well as flexibility, the spherical robot, for example, can move around inside a bending narrow pipeline slightly larger than its diameter. Even in the case when it has suffered from colliding against something else or falling down, it can restore stability by itself. This provides it with stronger viability than the traditional mobile robots, such as wheeled, legged and tracked ones[7-8]. Present research on spherical robot focuses mainly on designing mechanism with controllable and flexible movement and the nonholonomic control method with convergence and practicality.

Controllable as a nonholonomic system usually is, it does not satisfy the necessary condition required by the smooth state feedback law, so the traditional smooth static state feedback method can no longer be applied to it. Although different ways and means have been proposed to settle this problem, such as smooth time-varying stabilization, discontinuous time-independent stabilization, and hybrid stabilization[9], they are only fit for the nonholonomic systems that can be transformed into a chained form. Belonging to nonholonomic system notwithstanding, this can never be done with the spherical robot owing to its dissatisfaction with the necessary condition of differential smoothness required by chained system[10-12]. So the control problem of spherical robot can not be solved with methods proposed for chained form system. Although it has been demonstrated that the motion control of spherical robot can be solved with differential ge-

*Corresponding author. Tel.: +86-10-82317729.
E-mail address: qzhan@buaa.edu.cn
Foundation items: National Natural Science Foundation of China (50705003); National High Technology Research and Development Program of China (2007AA04Z252).

ometry[13], at present there is not an efficient method for it.

Historically, researches on motion control of spherical robot could be summarized as follows. Bicchi, et al. established a quasi-static kinematic model and a planar Lagrangian dynamic model of a spherical mobile robot, which were shown valid only under limited conditions[3]. Javadi, et al. considered the spherical robot as a chained system and established the dynamic model using Newton method and discussed the motion planning of the robot. However, simulations and experiments conducted in a smaller scale showed poor accuracy[4]. Mukherjee, et al. discussed the motion planning and proposed two open-loop control strategies for reconfiguration of rolling sphere[10]. Using dynamic theory of nonholonomic system, Sun, et al. analyzed the kinematic and dynamic problems of an omni directional spherical robot and found out the relationship between the maximal angle velocity and the pose of the robot[5,14]. Bhattacharya and Agrawal deduced a first-order mathematical model of a kind of spherical robots from the nonslip constraint and angular momentum conservation and presented simulation and experiment results[15]. Halme, et al. set up kinematic and dynamic models of a spherical mobile robot and analyzed the capabilities of the robot, such as uphill climbing and overrunning obstacles[16]. Cameron and Book discussed the kinematic and dynamic modeling of nonholonomic system and derived a simplified Boltzmann-Hamel equation for both holonomic and nonholonomic systems[17]. Zhan, et al. obtained the dynamic model of the spherical robot BHQ-1 on the basis of quasi-velocity and the simplified Boltzmann-Hamel equation. Simulations of linear and circular motions showed that this model could control the robot moving around at a high precision level in an interference-free environment. However, moving errors of the spherical robot will accumulate with time if there exists interference or control noise[18]. Li and Canny proved the controllability of a sphere with differential geometry and proposed a three-step motion control algorithm to converge its position and pose to the desired values[13]. Ripe as is the algorithm theoretically, it does not work if the spherical robot cannot turn with zero radius and nor does it if its motion is not so accurate as the algorithm demands. At present, almost all the control methods of spherical robots are open-looped, however, simulations and experiments show that it is difficult to achieve highly precise motion with open-loop methods and kinematic model.

This article tackles dynamic trajectory tracking of a spherical robot designed by our lab with a close-loop method and provides a trajectory tracking controller on the basis of the back-stepping method. The article is structured as follows. After a brief description of the structure and motion principle of spherical robot BHQ-2 in Section 2, its dynamic model is set up by using the simplified Boltzmann-Hamel equation in Section 3. Then the tracking control is discussed on the basis of kinematics and dynamics of the robot in Section 4, followed by designing the trajectory tracking controller using the back-stepping method in Section 5. This article will end up in proving convergence of the controller with the Lyapunov stability theory in Section 6 and presenting simulation results in Section 7.

2 Brief Introduction of Robot BHQ-2

The spherical robot BHQ-2 is mainly designed for unmanned environment exploration, for example, outer planets and desert. The first prototype of BHQ-2 was implemented in 2005. Fig.1 shows its 3D model and Fig.2 shows the prototype. BHQ-2 is composed of a two-wheeled car with a built-in motor, a motor, a hollow shaft, a heavy object (heavy shortly), and two cameras. The hollow shaft is connected to the shell through two ball bearings and serves as a frame where other components are installed. The two-wheeled car is fixed on the hollow shaft through a link with its wheels rolling on the inner surface of the shell. The motor that is driving the heavy is also fixed on the hollow shaft and tied to the heavy by a link. The motor can drive the heavy to swing around the motor shaft. Two cameras on the both ends of the hollow shaft can protrude from within to take environmental photos, which are transmitted to the control center through a wireless transmission system. On the basis of pictures, an operator is able to observe the environment and exercise remote control over the movements of the spherical robot.

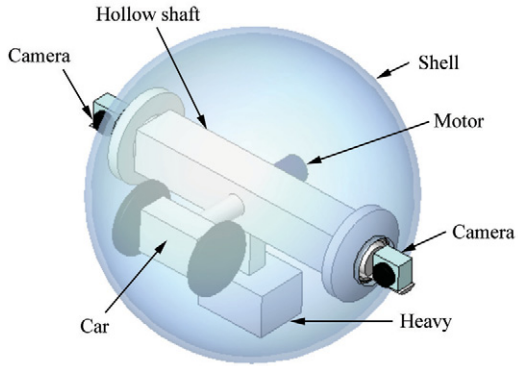

Fig.1 Structure of BHQ-2.

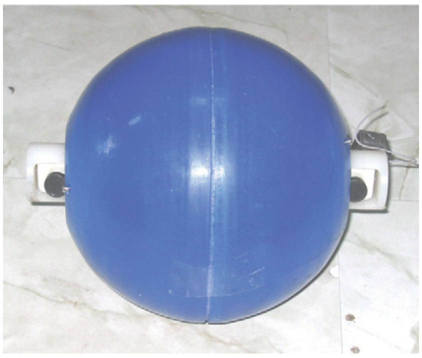

Fig.2 Prototype of BHQ-2.

When the car moves climbing up on the inner shell surface, it makes all the internal parts rotate, thus, causing the offset of the gravity center and creating a gravity moment, which moves the robot along a straight line. When the motor rotates, it makes the heavy rotate round the motor axis, thus, creating a side offset of the gravity moment to drive the robot to tilt aside. The combination of the two above-mentioned motions moves the spherical robot turning aside. As a result, the different motions of the spherical robot BHQ-2 can be easily achieved by joint control of the car and the motor.

3 Dynamic Modeling

As is shown by experiments, it is difficult for kinematics control to move a spherical robot along a given trajectory, so it is needed to establish the dynamic model of the robot before the tracking control problem is addressed. Although there have been a lot of methods to establish the dynamic model of nonholonomic robots, such as Gibbs-Appell equation, improved Lagrange equation, Kane equation, and Boltzmann-Hamel equation, it is almost impracticable for them to be used in this case because of the massive computational load and the complexity of the procedure. From D'Alembert-Lagrange equation

$$\sum_{k=1}^{n}\left(\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial T}{\partial \dot{q}_k}-\frac{\partial T}{\partial q_k}-Q_k\right)\delta q_k=0 \tag{1}$$

Cameron deduced a new simplified Boltzmann-Hamel equation[17]

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \overline{T}}{\partial \omega_I}+\sum_{j=1}^{n}\sum_{i=1}^{n}\eta_{iI}\gamma_{ij}\frac{\partial \overline{T}}{\partial \omega_j}-\sum_{j=1}^{n}\eta_{ji}\frac{\partial \overline{T}}{\partial q_j}=N_I \tag{2}$$

where $\boldsymbol{\omega}$ is the vector of quasi-velocities that has a linear relationship with derivatives of the generalized coordinates, $\overline{T}$ a new expression of the kinetic energy by replacing $\dot{q}$ in Eq.(1) with $\boldsymbol{\omega}$ in Eq.(2), N the generalized force, $\boldsymbol{\eta}$ a coefficient matrix and a function of the generalized coordinates, $\boldsymbol{\gamma}$ a coefficient matrix, I the independent quasi-coordinates (or related quantities), and n the number of the generalized coordinate q_j.

The simplified Boltzmann-Hamel Eq.(2) can be applied to both holonomic system and nonholonomic system. The coefficient γ can be easily calculated and the equation can be used more easily.

As shown in Fig.3, suppose the reference coordinates are $OXYZ$ and the robot coordinates are $O'X'Y'Z'$. With O' is the geometrical center of the spherical robot, axis X' is in line with the hollow axle, and axis Z' parallels with axis Z when BHQ-2 is at rest. Because the spherical robot BHQ-2

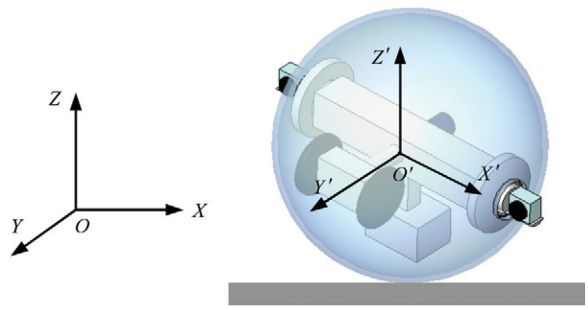

Fig.3 Coordinates setup of BHQ-2.

cannot move in Z direction, five variables (x, y, φ, β, ψ) are enough to describe its configuration. Let (x, y) be the coordinates of O' in the reference coordinates, and (φ, β, ψ) be the rotation angles around three coordinate axes, namely, X, Y, and Z, which denote the pose of the spherical robot BHQ-2.

For the spherical robot BHQ-2, φ and β can be determined directly by the control inputs, so the kinematics equation of BHQ-2 can be simplified into Eq.(3), of which the detailed deduction will be given in another article.

$$\left.\begin{aligned}\dot{x}&=r\dot{\varphi}\cos\beta\sin\psi+r\dot{\beta}\cos\psi\\ \dot{y}&=r\dot{\varphi}\cos\beta\cos\psi-r\dot{\beta}\sin\psi\\ \dot{\psi}&=\dot{\varphi}\sin\beta\end{aligned}\right\}\tag{3}$$

where r is the radius of the spherical robot.

In some cases, for simplicity, the velocity of a point is not expressed by the generalized velocities but by their linear forms (quasi-velocities), such as the kinetic energy. Normally, the nonholonomic constraint equations of a system can be chosen to be a part of the quasi-velocities, which are expected to facilitate calculation[19]. According to the kinematic equation of the spherical robot BHQ-2, five quasi-velocities ω_1, ω_2, ω_3, ω_4, ω_5 of the robot are chosen as follows

$$\left.\begin{aligned}\omega_1&=-\dot{\varphi}\cos\beta\cos\psi+\dot{\beta}\sin\psi\\ \omega_2&=\dot{\varphi}\cos\beta\sin\psi+\dot{\beta}\cos\psi\\ \omega_3&=\dot{\psi}+\dot{\varphi}\sin\beta\\ \omega_4&=\dot{x}-r\omega_2\\ \omega_5&=\dot{y}+r\omega_1\end{aligned}\right\}\tag{4}$$

From Eq.(4), the kinematic equation of BHQ-2 in terms of quasi-velocities can be written by

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\psi}\\ \dot{\varphi}\\ \dot{\beta}\end{bmatrix}=\begin{bmatrix}0 & r & 0 & 1 & 0\\ -r & 0 & 0 & 0 & 1\\ \cos\psi\tan\beta & -\sin\psi\tan\beta & 1 & 0 & 0\\ -\cos\psi\sin\beta & \sin\psi\sin\beta & 0 & 0 & 0\\ \sin\psi & \cos\psi & 0 & 0 & 0\end{bmatrix}\cdot\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\\ \omega_4\\ \omega_5\end{bmatrix}\tag{5}$$

From Eq.(2), Eq.(5), kinetic energy and potential energy of the spherical robot BHQ-2, a very complex dynamic model of the spherical robot can be derived and expressed in the following simplified form.

$$\boldsymbol{M}(q)\dot{\boldsymbol{\Omega}}+\boldsymbol{C}(q,\dot{q})\boldsymbol{\Omega}+\boldsymbol{N}(q,\dot{q})=\boldsymbol{B}(q)\boldsymbol{\Gamma}\tag{6}$$

where $\boldsymbol{M}(q)$ is a matrix of positive and symmetrical inertia, $\boldsymbol{C}(q,\dot{q})$ a matrix of coriolis and centripetal force, $\boldsymbol{\Omega}$ a matrix of quasi-velocities, $\boldsymbol{N}(q,\dot{q})$ a matrix of gravity and friction, $\boldsymbol{B}(q)$ a transformation matrix of input, and $\boldsymbol{\Gamma}$ a matrix of the projects of driving torque on each coordinate axis.

4 Tracking Control Problem

For simplicity, Eq.(3) can be rewritten into Eq.(7) and Eq.(6) into Eq.(8).

$$\left.\begin{aligned}\dot{x}&=u\sin\psi+v\cos\psi\\ \dot{y}&=u\cos\psi-v\sin\psi\\ \dot{\psi}&=\omega\end{aligned}\right\}\tag{7}$$

where $u=r\dot{\varphi}\cos\beta$, $v=r\dot{\beta}$, and $\omega=\dot{\psi}$.

$$\left.\begin{aligned}\dot{u}&=c_{11}v\omega-c_{12}u+c_{13}\tau_u\\ \dot{v}&=-c_{21}u\omega-c_{22}v+c_{23}\tau_v\\ \dot{\omega}&=c_{31}(q)uv-c_{32}\omega\end{aligned}\right\}\tag{8}$$

where $c_{ij}(1\le i\le 2,1\le j\le 3)$ are constants, c_{3j} $(1\le j\ge 2)$ coefficients varying with the configuration of the robot, τ_u and τ_v the driving torque, $c_{11}v\omega$, $c_{21}u\omega$, $c_{31}uv$ coriolis acceleration.

Suppose the current configuration vector of the spherical robot is $\boldsymbol{p}=(x,y,\psi)^{\mathrm{T}}$ and the desired configuration vector $\boldsymbol{p}_{\mathrm{d}}=(x_{\mathrm{d}},y_{\mathrm{d}},\psi_{\mathrm{d}})^{\mathrm{T}}$ (see Fig.4).

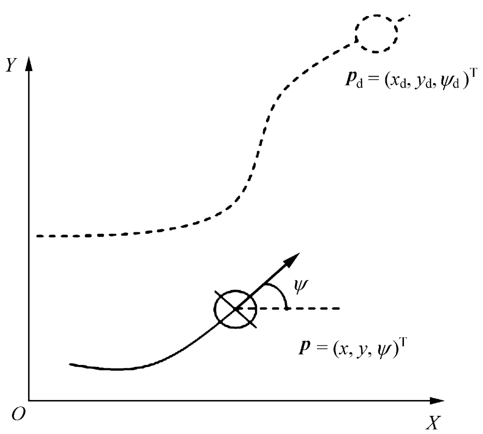

Fig.4 Sketch of tracking problem.

The configuration error vector of the robot, $\boldsymbol{p}_{\mathrm{e}}=(x_{\mathrm{e}},y_{\mathrm{e}},\psi_{\mathrm{e}})^{\mathrm{T}}$, can be expressed by

$$\boldsymbol{p}_{\mathrm{e}}=\begin{bmatrix}x_{\mathrm{e}}\\ y_{\mathrm{e}}\\ \psi_{\mathrm{e}}\end{bmatrix}=\begin{bmatrix}\cos\psi & \sin\psi & 0\\ -\sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}x-x_{\mathrm{d}}\\ y-y_{\mathrm{d}}\\ \psi-\psi_{\mathrm{d}}\end{bmatrix}\tag{9}$$

Define the tracking velocity errors as $u_{\mathrm{e}}=$

$u-u_{\mathrm{d}}$, $v_{\mathrm{e}}=v-v_{\mathrm{d}}$, $\omega_{\mathrm{e}}=\omega-\omega_{\mathrm{d}}$. By differentiating Eq.(9), can be obtained

$$\left.\begin{aligned} &\dot{x}_{\mathrm{e}}=u_{\mathrm{e}}-u_{\mathrm{d}}(\cos\psi_{\mathrm{e}}-1)-v_{\mathrm{d}}\sin\psi_{\mathrm{e}}+\omega y_{\mathrm{e}}\\ &\dot{y}_{\mathrm{e}}=v_{\mathrm{e}}-v_{\mathrm{d}}(\cos\psi_{\mathrm{e}}-1)+u_{\mathrm{d}}\sin\psi_{\mathrm{e}}-\omega x_{\mathrm{e}}\\ &\dot{\psi}_{\mathrm{e}}=\omega_{\mathrm{e}}\\ &\dot{u}_{\mathrm{e}}=c_{11}v\omega+c_{12}u+c_{13}\tau_u-\dot{u}_{\mathrm{d}}\\ &\dot{v}_{\mathrm{e}}=-c_{21}u\omega-c_{22}v+c_{23}\tau_v-\dot{v}_{\mathrm{d}}\\ &\dot{\omega}_{\mathrm{e}}=\dot{\omega}-\dot{\omega}_{\mathrm{d}} \end{aligned}\right\}\tag{10}$$

Thus the tracking control problem turns into the stabilization problem in the above error system[20]. Here the control inputs are τ_u and τ_v corresponding to the inputs of the two motors of the robot. However, it is found difficult to design the controller for the control inputs τ_u and τ_v . Now let the relationship between the velocity $\dot{\psi}$ and the angle β be $\dot{\psi}=u\tan\beta/r$ and by differentiating we can acquire

$$\dot{\omega}=\frac{\dot{u}\tan\beta+u\sec^2\beta\cdot\dot{\beta}}{r}=\frac{r\dot{u}\tan\beta+u\sec^2\beta\cdot v}{r^2}\tag{11}$$

From Eq.(11), the following equation can be achieved, i.e., $v=\dfrac{r^2\dot{\omega}-r\dot{u}\tan\beta}{u\sec^2\beta}$.

Using the torque τ_u and the angular acceleration $\dot{\omega}$ as control inputs to design the controller, the roll velocity v can be attained with the above equation. At last, τ_u and v are used as the inputs to control the spherical robot.

Before designing the controller, suppose that the desired trajectory satisfies the following assumptions:

Assumption 1

(1) The reference signals x_{d}, y_{d}, ψ_{d}, u_{d}, v_{d}, ω_{d}, $\dot{u}_{\mathrm{d}}$, $\dot{\omega}_{\mathrm{d}}$, and $\ddot{u}_{\mathrm{d}}$ are bounded.

(2) One of the following condition holds:

(a) There exists such a positive constant σ_{r} that, for any pair of $(t_0,t), 0\le t_0\le t<\infty$,

$$\int_{t_0}^{t}\omega_{\mathrm{d}}^2(t)\mathrm{d}t\ge\sigma_{\mathrm{r}}(t-t_0)\tag{12}$$

(b) There exist $x_1\ge0$ and $x_2\ge0$ and

$$\left.\begin{aligned} &0<\sigma_{u\min}\le|u_{\mathrm{d}}(t)|\\ &|\omega_{\mathrm{d}}(t)|\le x_1e^{-x_2(t-t_0)},\quad t_0\in[0,t] \end{aligned}\right\}\tag{13}$$

The above assumptions guarantee at least either $u_{\mathrm{d}}(t)$ or $\omega_{\mathrm{d}}(t)$ not approaching zero and the sign of u_{d} remaining unchanged. When both $u_{\mathrm{d}}(t)$ and $\omega_{\mathrm{d}}(t)$ equal zero, the tracking problem turns into the stabilization one that can never be solved by any time-independent smooth feedback[21].

5 Controller Design

To begin with, the virtual velocity control is designed for inputs u_{e} and ω_{e} so as to globally asymptotically stabilize x_{e}, y_{e}, ψ_{e}, and v_{e} to the origin. On the basis of the back-stepping method, the control inputs τ_u and $\dot{\omega}$ are so designed as to make the errors between the virtual velocity control quantities and the actual ones exponentially approximate to zero.

Define the coordinate transformation as

$$z_{\mathrm{e}}=\psi_{\mathrm{e}}+a\sin\left(\frac{ku_{\mathrm{d}}y_{\mathrm{e}}}{\sqrt{1+x_{\mathrm{e}}^2+y_{\mathrm{e}}^2}}\right)\tag{14}$$

where k is a positive constant that satisfies $ku_{\mathrm{d}\max}\le k^*$ and later is chosen $k^*<1$ in the stability analysis; $u_{\mathrm{d}\max}$ the maximum values of $|u_{\mathrm{d}}|$. It can be seen that z_{e} converges with the convergence of the error system. It can also be noticed that using Eq.(14) in place of $z_{\mathrm{e}}=\psi_{\mathrm{e}}+ky_{\mathrm{e}}$ can prevent the spherical robot from severe shifts to right or to left when y_{e} becomes bigger[22]. Through transformation, the error equations of the spherical robot can be rewritten into

$$\left.\begin{aligned} &\dot{x}_{\mathrm{e}}=u+\omega y_{\mathrm{e}}-(u_{\mathrm{d}}\cos z_{\mathrm{e}}+v_{\mathrm{d}}\sin z_{\mathrm{e}})\bar{\omega}_{\mathrm{e}}-\\ &\qquad(u_{\mathrm{d}}\sin z_{\mathrm{e}}-v_{\mathrm{d}}\cos z_{\mathrm{e}})ku_{\mathrm{d}}y_{\mathrm{e}}\bar{\omega}_{\mathrm{e1}}\\ &\dot{y}_{\mathrm{e}}=v-\omega x_{\mathrm{e}}-(v_{\mathrm{d}}\cos z_{\mathrm{e}}+u_{\mathrm{d}}\sin z_{\mathrm{e}})\bar{\omega}_{\mathrm{e}}-\\ &\qquad(v_{\mathrm{d}}\sin z_{\mathrm{e}}-u_{\mathrm{d}}\cos z_{\mathrm{e}})ku_{\mathrm{d}}y_{\mathrm{e}}\bar{\omega}_{\mathrm{e1}}\\ &\dot{z}_{\mathrm{e}}=(1-ku_{\mathrm{d}}x_{\mathrm{e}}/\bar{\omega}_{\mathrm{e2}})\omega_{\mathrm{e}}+1/\bar{\omega}_{\mathrm{e2}}\cdot(f_z+f_u)-\\ &\qquad ku_{\mathrm{d}}x_{\mathrm{e}}y_{\mathrm{e}}\bar{\omega}_{\mathrm{e1}}^2(u_{\mathrm{e}}+k_1x_{\mathrm{e}}-k_2\omega_{\mathrm{d}}y_{\mathrm{e}})/\bar{\omega}_{\mathrm{e2}}\\ &\dot{u}_{\mathrm{e}}=\dot{u}-\dot{u}_{\mathrm{d}}\\ &\dot{v}_{\mathrm{e}}=\dot{v}-\dot{v}_{\mathrm{d}}\\ &\dot{\omega}_{\mathrm{e}}=\dot{\omega}-\dot{\omega}_{\mathrm{d}} \end{aligned}\right\}\tag{15}$$

where

$$\left.\begin{aligned}
f_x &= -(u_\mathrm{d}\cos z_\mathrm{e} + v_\mathrm{d}\sin z_\mathrm{e} - u_\mathrm{d})\bar{\omega}_\mathrm{e} - \\
&\quad (u_\mathrm{d}\sin z_\mathrm{e} - v_\mathrm{d}\cos z_\mathrm{e} + v_\mathrm{d})ku_\mathrm{d}y_\mathrm{e}\bar{\omega}_\mathrm{e1} \\
f_y &= -(v_\mathrm{d}\cos z_\mathrm{e} - u_\mathrm{d}\sin z_\mathrm{e} - v_\mathrm{d})\bar{\omega}_\mathrm{e} - \\
&\quad (v_\mathrm{d}\sin z_\mathrm{e} + u_\mathrm{d}\cos z_\mathrm{e} - u_\mathrm{d})ku_\mathrm{d}y_\mathrm{e}\bar{\omega}_\mathrm{e1} \\
f_z &= ku_\mathrm{d}f_y - ku_\mathrm{d}y_\mathrm{e}(x_\mathrm{e}f_x + y_\mathrm{e}f_y)\bar{\omega}_\mathrm{e1}^2 \\
f_u &= ku_\mathrm{d}v_\mathrm{e} + ku_\mathrm{d}v_\mathrm{d}(1-\bar{\omega}_\mathrm{e}) - k^2u_\mathrm{d}^3y_\mathrm{e}\bar{\omega}_\mathrm{e1} - \\
&\quad ku_\mathrm{d}\omega_\mathrm{d}x_\mathrm{e} + k\dot{u}_\mathrm{d}y_\mathrm{e} - ku_\mathrm{d}\bar{\omega}_\mathrm{e1}^2y_\mathrm{e}(x_\mathrm{e}u_\mathrm{e}^\mathrm{d} + \\
&\quad x_\mathrm{e}u_\mathrm{d} - x_\mathrm{e}u_\mathrm{d}\bar{\omega}_\mathrm{e} + ku_\mathrm{d}v_\mathrm{d}\bar{\omega}_\mathrm{e1}x_\mathrm{e}y_\mathrm{e} + \\
&\quad y_\mathrm{e}v - y_\mathrm{e}v_\mathrm{d}\bar{\omega}_\mathrm{e} - ku_\mathrm{d}^2\bar{\omega}_\mathrm{e1}x_\mathrm{e}y_\mathrm{e}^2) \\
\bar{\omega}_\mathrm{e1} &= \sqrt{\frac{1}{1+x_\mathrm{e}^2+y_\mathrm{e}^2}} \\
\bar{\omega}_\mathrm{e2} &= \sqrt{1+x_\mathrm{e}^2+[1+(ku_\mathrm{d})^2]y_\mathrm{e}^2} \\
\bar{\omega}_\mathrm{e} &= \bar{\omega}_\mathrm{e1}\bar{\omega}_\mathrm{e2}
\end{aligned}\right\} \tag{16}$$

The controller will be designed by the following two steps.

Step 1:

Define the virtual control errors as

$$\tilde{u}_\mathrm{e} = u_\mathrm{e} - u_\mathrm{e}^\mathrm{d},\ \tilde{\omega}_\mathrm{e} = \omega_\mathrm{e} - \omega_\mathrm{e}^\mathrm{d}$$

where u_e^d and $\omega_\mathrm{e}^\mathrm{d}$ denote the virtual velocity control quantities of u_e and ω_e, which are chosen to be

$$\left.\begin{aligned}
u_\mathrm{e}^\mathrm{d} &= -k_1x_\mathrm{e} + k_2\omega_\mathrm{d}y_\mathrm{e} \\
\omega_\mathrm{e}^\mathrm{d} &= \omega_\mathrm{1e}^\mathrm{d} + \omega_\mathrm{2e}^\mathrm{d}
\end{aligned}\right\} \tag{17}$$

where

$$\omega_\mathrm{1e}^\mathrm{d} = -\frac{f_u}{\bar{\omega}_\mathrm{e2} - ku_\mathrm{d}x_\mathrm{e}},\ \omega_\mathrm{2e}^\mathrm{d} = -\frac{k_3\bar{\omega}_\mathrm{e2}z_\mathrm{e} + f_z}{\bar{\omega}_\mathrm{e2} - ku_\mathrm{d}x_\mathrm{e}}$$

with k_i (i=1,2,3) are positive constants. Here $\omega_\mathrm{e}^\mathrm{d}$ is defined as the sum of $\omega_\mathrm{1e}^\mathrm{d}$ and $\omega_\mathrm{2e}^\mathrm{d}$ in order to ease the later calculation and analysis. In Eq.(17), the first part of the right expression of u_e^d is used to stabilize x_e and the second y_e.

Step 2:

By differentiating the virtual control errors with respect to the solution of Eq.(15) and Eq.(17), the following can be acquired, i.e.,

$$\left.\begin{aligned}
\dot{\tilde{u}}_\mathrm{e} &= \dot{u} - \dot{u}_\mathrm{d} - \dot{u}_\mathrm{e}^\mathrm{d} \\
\dot{\tilde{\omega}}_\mathrm{e} &= \dot{\omega} - \dot{\omega}_\mathrm{d} - \dot{\omega}_\mathrm{e}^\mathrm{d}
\end{aligned}\right\} \tag{18}$$

where $\dot{u}_\mathrm{e}^\mathrm{d}$ and $\dot{\omega}_\mathrm{e}^\mathrm{d}$ are the first derivatives of u_e^d and $\omega_\mathrm{e}^\mathrm{d}$ with respect to the solution of Eq.(15).

From Eq.(18) the actual control inputs τ_u and v are chosen to be

$$\left.\begin{aligned}
\tau_u &= \frac{1}{c_{13}}[-k_4\tilde{u}_\mathrm{e} - c_{11}v\omega + c_{12}(u_\mathrm{e}^\mathrm{d} + u_\mathrm{d}) + \dot{u}_\mathrm{d} - \dot{u}_\mathrm{e}^\mathrm{d}] \\
v &= \frac{r^2\dot{\omega} - r\dot{u}\tan\beta}{u\sec^2\beta}
\end{aligned}\right\} \tag{19}$$

where $\dot{\omega} = c_{32}(\omega_\mathrm{e}^\mathrm{d} - \omega_\mathrm{e}) - k_5\tilde{\omega}_\mathrm{e} + \dot{\omega}_\mathrm{d} + \dot{\omega}_\mathrm{e}^\mathrm{d}$, $\dot{u} = c_{11}\cdot v\omega - c_{12}u + c_{13}\tau_u$, k_4 and k_5 are positive constants to be chosen later.

If the Assumption 1 is satisfied, the above controller will allow the spherical robot to achieve global and asymptotical tracking of the desired trajectory by proper selection of the coefficients k and k_i, $1 \le i \le 5$.

6 Convergence of the Controller

Next, the problem about convergence of the robot configuration errors and the virtual control errors will be discussed. Combing Eq.(19) and Eq.(15), a close-loop system can be obtained.

$$\dot{\boldsymbol{X}}_\mathrm{e} = f(t, \boldsymbol{X}_\mathrm{e}) \tag{20}$$

where $\boldsymbol{X}_\mathrm{e} = [x_\mathrm{e}\quad y_\mathrm{e}\quad z_\mathrm{e}\quad \tilde{u}_\mathrm{e}\quad \tilde{\omega}_\mathrm{e}]^\mathrm{T}$

$$f(t,\boldsymbol{X}_\mathrm{e}) = \begin{bmatrix}
-k_1x_\mathrm{e} + k_2\omega_\mathrm{d}y_\mathrm{e} + u_\mathrm{d}(1-\bar{\omega}_\mathrm{e}) + ku_\mathrm{d}v_\mathrm{d}\bar{\omega}_\mathrm{e1}y_\mathrm{e} + \omega y_\mathrm{e} + f_x + \tilde{u}_\mathrm{e} \\
v_\mathrm{e} + v_\mathrm{d}(1-\bar{\omega}_\mathrm{e}) - ku_\mathrm{d}^2\bar{\omega}_\mathrm{e1}y_\mathrm{e} - \omega_\mathrm{e}x_\mathrm{e} - \omega_\mathrm{d}x_\mathrm{e} + f_y \\
-k_3z_\mathrm{e} + (1 - ku_\mathrm{d}x_\mathrm{e}/\bar{\omega}_\mathrm{e2})\bar{\omega}_\mathrm{e} - ku_\mathrm{d}x_\mathrm{e}y_\mathrm{e}\bar{\omega}_{e1}^2\tilde{u}_\mathrm{e}/\bar{\omega}_\mathrm{e2} \\
-(k_6 + c_{12})\bar{u}_\mathrm{e} \\
-(k_7 + c_{32})\bar{\omega}_\mathrm{e}
\end{bmatrix} \tag{21}$$

To prove the convergence of the system, the following Lyapunov function is introduced.

$$V = \frac{1}{2}(x_\mathrm{e}^2 + y_\mathrm{e}^2 + z_\mathrm{e}^2 + \tilde{u}_\mathrm{e}^2 + \tilde{\omega}_\mathrm{e}^2) \tag{22}$$

Differentiating V with respect to $\dot{\boldsymbol{X}}_\mathrm{e} = f(t,\boldsymbol{X}_\mathrm{e})$, the following can be attained

$$\dot{V} = -k_1x_\mathrm{e}^2 - ku_\mathrm{d}^2\bar{\omega}_\mathrm{e1}y_\mathrm{e}^2 + k_2\omega_\mathrm{d}x_\mathrm{e}y_\mathrm{e} + u_\mathrm{d}x_\mathrm{e}(1-\bar{\omega}_\mathrm{e}) +$$

$$ku_{\rm d}v_{\rm d}\bar{\omega}_{\rm e1}x_{\rm e}y_{\rm e}-k_3z_{\rm e}^2+(1-ku_{\rm d}x_{\rm e}/\bar{\omega}_{\rm e2})\tilde{\omega}_{\rm e}z_{\rm e}-$$
$$ku_{\rm d}x_{\rm e}y_{\rm e}\bar{\omega}_{\rm e1}^2\tilde{u}_{\rm e}z_{\rm e}/\bar{\omega}_{\rm e2}-(k_4+c_{12})\tilde{u}_{\rm e}^2-(k_5+c_{32})\tilde{\omega}_{\rm e}^2 \tag{23}$$

Substituting $\bar{\omega}_{\rm e}$, $\bar{\omega}_{\rm e1}$, $\bar{\omega}_{\rm e2}$, $\omega_{\rm e}^{\rm d}$, $\omega_{\rm 1e}^{\rm d}$ and $\omega_{\rm 2e}^{\rm d}$ into Eq.(13), we have

$$\dot{V}\le-\mu_1(t)x_{\rm e}^2-\mu_{21}(t)y_{\rm e}^2-$$
$$\mu_{22}(t)y_{\rm e}^2/\sqrt{1+x_{\rm e}^2+y_{\rm e}^2}-\rho(z_{\rm e}^2+\tilde{u}_{\rm e}^2+\tilde{\omega}_{\rm e}^2) \tag{24}$$

where

$$\mu_1(t)=k_1-\frac{1}{4\varepsilon_1}\left(k_2+ku_{\rm d}^2+kv_{\rm d}^2\right)$$
$$\mu_{21}(t)=k_2\varepsilon_1\omega_{\rm d}^2$$
$$\mu_{22}(t)=\left(k-2k\varepsilon_1\right)u_{\rm d}^2$$
$$\rho_2=\min\{(k_3-1.5),(k_4+c_{12}-0.5),(k_5+c_{32}-1)\}$$

In order to make ρ_2 positive, it is necessary to assume the design constants k_3, k_4, and k_5 to be >1.5, >0.5, and >1.0, respectively, and ε_i (i=1,2,3) are positive constants. $u_1(t)$, $u_{21}(t)$, and $u_{22}(t)$ are functions of time varying with the time-dependent reference velocities. Hence, if the coefficients k, k_1, and k_2 satisfy the following conditions:

(a) $ku_{\rm d\max}\le k^*<1,\ u_1(t)\ge\mu_1^*>0,\ \forall t\ge 0$.

(b) $k-2k\varepsilon_1\ge 0, k_2\varepsilon_1\sigma_r\ge\mu_{21}^*>0$ under the condition Eq.(12) in Assumption 1.

(c) $\mu_{22}(t)\ge\mu_{22}^*>0$ under the condition Eq.(13) in Assumption 1.

where $\omega_{\rm dmax}$ is the maximum value of $|\omega_{\rm d}(t)|$. Then, the following conclusion could be reached.

(1) If the condition (a) is satisfied, then,

$$\dot{V}\le-\mu_1^*x_{\rm e}^2-k_2\varepsilon_1\omega_{\rm d}^2y_{\rm e}^2-\rho(z_{\rm e}^2+\tilde{u}_{\rm e}^2+\tilde{\omega}_{\rm e}^2) \tag{25}$$

(2) If the condition (b) is satisfied, then,

$$\dot{V}\le-\mu_1^*x_{\rm e}^2-\mu_{22}^*y_{\rm e}^2/\sqrt{1+x_{\rm e}^2+y_{\rm e}^2}-$$
$$k_2\varepsilon_1\chi_1\chi_1e^{-2\chi_2(t-t_0)}y_{\rm e}^2-\rho(z_{\rm e}^2+\tilde{u}_{\rm e}^2+\tilde{\omega}_{\rm e}^2) \tag{26}$$

It can be seen that the system is asymptotically stabilized in both conditions.

Consequently, it is demonstrated that the spherical robot succeeds in achieving global asymptotical tracking of the desired trajectory.

7 Simulations

Numerical simulations on the spherical robot BHQ-2 are performed to validate the effectiveness of the trajectory tracking controller. The discussions are carried out separately on tracking of a linear trajectory and that of a circular trajectory. Let the diameter of BHQ-2 be 200 mm.

(1) Tracking of a linear trajectory

Suppose the desired trajectory to be

$$\left.\begin{aligned}x=0.5t\\y=0.5t\end{aligned}\right\} \tag{27}$$

The initial configuration of the spherical robot is $x_0 = 0$, $y_0 = 2$, $\psi_0 = 0$, and $\beta_0 = 0$. Now that condition Eq.(13) is satisfied, the coefficients are selected to be $k = 2$, $k_1 = 2$, $k_2 = 0.5$, $k_3 = 2$, $k_4 = 5$, $k_5 = 5$. Fig.5 and Fig.6 illustrate the simulation results.

From the above simulation, it can be concluded that the spherical robot with the designed controller is capable of tracking a desired linear trajectory.

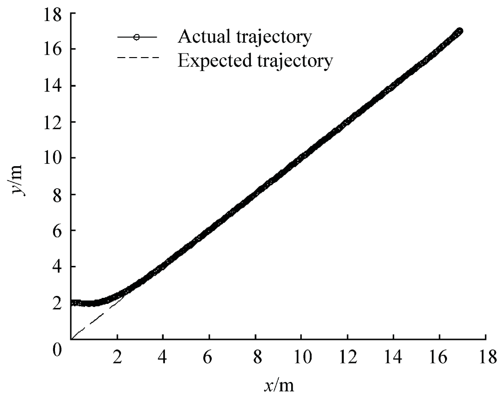

Fig.5 Simulation of tracking of a linear trajectory.

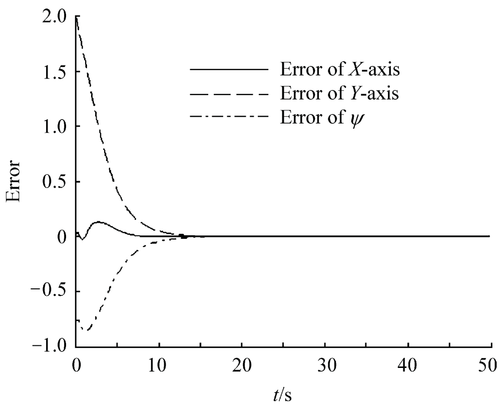

Fig.6 Configuration errors.

(2) Tracking of a circular trajectory

Suppose the desired circular trajectory to be

$$\left.\begin{aligned}x=\cos(0.2t)\\y=\sin(0.2t)\end{aligned}\right\} \tag{28}$$

The initial configuration of the spherical robot is $x_0 = 0$, $y_0 = 0$, $\psi_0 = \pi/2$, and $\beta_0 = 0$. Now that the condition Eq.(12) is satisfied, the coefficients are selected to be $k = 2$, $k_1 = 0.4$, $k_2 = 0.5$, $k_3 = 2$, $k_4 = 5$, $k_5 = 5$. Fig.7 and Fig.8 illustrate the simulation results.

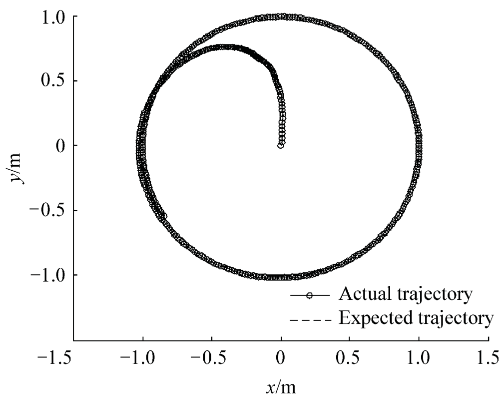

Fig.7 Simulation of tracking a circular trajectory.

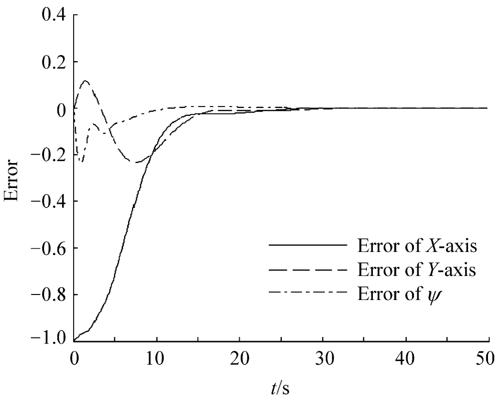

Fig.8 Configuration errors.

From the simulation, it is concluded that the spherical robot with the designed controller is capable of globally asymptotically tracking a desired circular trajectory.

8 Conclusions

This article has made a brief description of the structure and motion principle of the spherical mobile robot BHQ-2 designed for unmanned environment exploration. The dynamic model of BHQ-2 is firstly represented by a simplified Boltzmann-Hamel equation, and then taking into account its dynamic properties, a trajectory tracking controller of the robot is designed on the basis of the back-stepping method. Its convergence is verified by Lyapunov's function. Numerical simulations show that, by means of the designed controller, a successfully global and asymptotical tracking of desired trajectories, either linear or circular, can be realized on the spherical robot BHQ-2. Besides, this controller presents easier requirements for the constraints to which the desired trajectory is the subject.

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Biographies:

Zhan Qiang He received the bachelor's and doctoral degrees in mechanical and electronics engineering from Harbin Institute of Technology, Harbin, China, in 1995 and 1999, respectively. From 1999 to 2001, he was a postdoctoral research fellow in the Robotics Institute, Beijing University of Aeronautics and Astronautics (BUAA), Beijing, China, and then became an associate professor. In 2007, he worked as a visiting scholar in University of Cassino, Cassino, Italy. His main research interests include spherical mobile robot, dexterous hand and robotic gripper, multi-rotor flying robot, multi-robot system and robot vision.
E-mail: qzhan@buaa.edu.cn

Liu Zengbo He received the bachelor's degree in mechanical engineering from Shandong University, Ji'nan, China, in 2005. He is currently working toward the master's degree in mechanical and electronics engineering at Robotics Institute, BUAA, Beijing, China. His current research interests include movement control of spherical mobile robot and DSP control of DC motors.
E-mail: liuzengbo@me.buaa.edu.cn

Cai Yao He received bachelor's degree in automobile engineering from BUAA, Beijing, China, in 2006. He is pursuing the doctoral degree in mechanical engineering at the Robotics Institute of BUAA. His current research interests include movement control of spherical robot and robotic control system design based on ARM and DSP.
E-mail: caiyao@me.buaa.edu.cn